MITSUBISHI ELECTRIC
Changes for the Better

One-Round Authenticated Group Key Exchange from Isogenies

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Katsuyuki TAKASHIMA (Mitsubishi Electric)

Joint work with Atsushi FUJIOKA (Kanagawa U.),
Shintaro TERADA, Kazuki YONEYAMA (Ibaraki U.)

Agenda

- Background on Isogeny-Based Cryptography
  - Isogenies between Elliptic Curves
  - HHS, CIM and CSIDH Key Exchange
- One-Round Authenticated Group Key Exchange (AGKE) on CIM
  - Proposed Protocols: n-UM and Biclique n-DH Protocols
  - Realistic 2-Party AKE from HHS (e.g., CSIDH)
- One-Round Authenticated Key Exchange (AKE) from SIDH
  - SIDH UM and Biclique Protocols
  - Security of Biclique SIDH AKE from di-SI-GDH Assumption

Background on Isogeny-Based Cryptography

Threat of Quantum Computers and Candidate Post-Quantum PKE

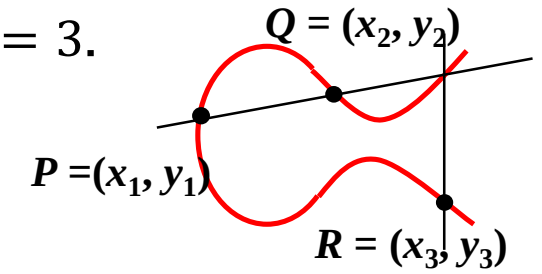
- P. Shor's factoring and discrete logarithm quantum algorithms break widely-used public key encryption (PKE), e.g., RSA, (EC)DH, Pairing crypto.
- Candidates of post-quantum PKE
  - lattice crypto
  - multivariate crypto
  - code-based crypto
  - isogeny crypto
- Quantum-resistant basic mathematical problems
  - lattice crypto: shortest vector problem (SVP), closest vector problem (CVP)
  - isogeny crypto: (SIDH) isogeny problem, (CSIDH) group action inversion problem

Elliptic Curves in Cryptography

- An elliptic curve E over a finite field $\mathbb{F}_q$ $(q = p^k,\ p \geq 5)$ is defined by an equation $y^2 = f(x)$ s.t. $\deg(f) = 3$.

  E.g., Montgomery form

  $E:\ by^2 = x^3 + ax^2 + x, \quad a, b(\neq 0) \in \mathbb{F}_q$



- $R := P + Q,\ \ (x_3, y_3) := (x_1, y_1) + (x_2, y_2)$ $\quad x_3 := b\{(y_2-y_1)/(x_2-x_1)\}^2-x_1-x_2-a$, $y_3 := \{(y_2-y_1)/(x_2-x_1)\}(x_1-x_3)-y_1$

- $\alpha \cdot P := \underbrace{P + \ldots + P}_{\alpha}$ $\longleftrightarrow$ g^{α} in the Diffie–Hellman key exchange

- ECDH : based on the hardness of calculating α from $(P, \alpha \cdot P)$: EC discrete logarithm

Isogenies between Elliptic Curves

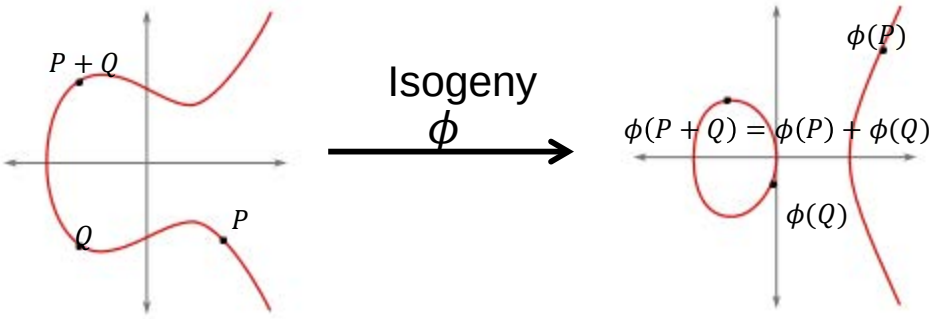
- Given two elliptic curves E, E', an isogeny $\phi: E \to E'$ is a surjective map defined on points $P = (x, y)$ of E by

  $\phi(P) = (f(x,y), g(x,y))$ where f, g are rational functions.

  Additionally, if P, Q are points on E, then

  $\phi(P+Q) = \phi(P) + \phi(Q)$ i.e., ϕ is a homomorphism.

- If there is an isogeny between two elliptic curves E, E', they are called isogenous.



- Vélu's formula: Using elliptic curve E and point R, efficiently compute an isogeny $\phi: E \to E/\langle R \rangle$ with kernel $\langle R \rangle$.

Algorithms for Isogeny Problems

Isogeny Problem

Given two isogenous elliptic curves E and E', compute a (compact representation of) isogeny $\phi: E \to E'$.

- For example, a compact representation of ϕ is given by elliptic curves (j-invariants) appearing in the sequence of ℓ_i-isogenies if ϕ is decomposed into a product of ℓ_i-isogenies for small prime ℓ_i's.

	Computation time with classical computers	Computation time with quantum computers
ordinary elliptic curves	$\tilde{O}(\sqrt[4]{p})$	$L_p[1/2, \sqrt{3}/2]$
supersingular elliptic curve *	$\tilde{O}(\sqrt{p})$	$\tilde{O}(\sqrt[4]{p})$

* elliptic curves E with $E[p] = \{O_E\}$

subexponental in log p: $L_p[\alpha, c] := \exp\big((c + o(1))(\log p)^{\alpha}(\log\log p)^{1-\alpha}\big)$

Hard Homogenous Spaces (HHS) [Cou97,06]

- A HHS (G, X) consists of
  - a finite commutative group G and
  - some set X (not necessarily a group), where
  - G acts freely and transitively on X, i.e., $G \curvearrowright X$

- The following tasks are required to be easy (e.g., polynomial-time)
  - compute the group operations on G,
  - sample randomly from G with (close to) uniform distribution,
  - decide validity and equality of a representation of elements of X,
  - compute the action of a group element $g \in G$ on some $x \in X$.

HHS-Based DH Protocol

- Fundamental intractability assumptions on HHS (G, X)
  - Inversion intractability of the group action:
    Intractable to compute g from $(x, g * x)$
  - 2-way computational DH (2-CDH) assumption:
    Intractable to compute $(gh) * x$ from $(x, g * x, h * x)$
  - variants of 2-CDH: 2-DDH, 2-GDH assumptions

- Standard HHS-based DH protocol:

Alice

$x \xleftarrow{U} X$, public parameter

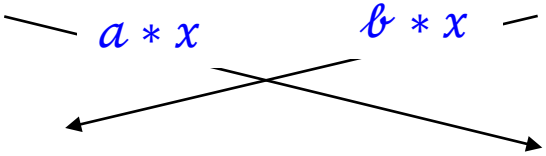
Bob

$a \xleftarrow{U} G$, Alice's secret key,

compute $a * x$,

$b \xleftarrow{U} G$, Bob's secret key,

compute $b * x$,

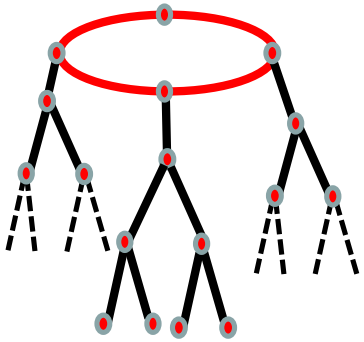


$K_{\text{Alice}} := a * (b * x)$

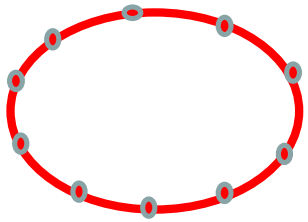
$K_{\text{Bob}} := b * (a * x)$

Examples of HHS

- CRS (Couveignes-Rostovtsev-Stolbunov) HHS (G, X)
  - $X := \mathrm{Ell}_q(\mathcal{O})$: the set of isomorphism classes over $\overline{\mathbb{F}}_q$ of ordinary elliptic curves with CM by $\mathcal{O}$ (an imaginary quadratic order)
  - $G := \mathrm{Cl}(\mathcal{O})$: the ideal class group of $\mathcal{O}$



- CSIDH (by [CLMPR18]) HHS (G, X)
  - X: the set of isomorphism classes over $\mathbb{F}_p$ of supersingular elliptic curves with $\mathbb{F}_p$-rational endomorphism ring by $\mathcal{O}$ (an imaginary quadratic order)
  - $G := \mathrm{Cl}(\mathcal{O})$: the ideal class group of $\mathcal{O}$



  - Same as the CRS-based key exchange, CSIDH has a subexponential-time quantum algorithm attack.
  - Efficient public key validation

Useful Applications from Isogenies ?

- Very recently, Boneh et al. suggest another research direction for using isogenies on elliptic curves in applied cryptography which includes:
  - n-way non-interactive key exchange,
  - verifiable random functions,
  - constrained pseudorandom functions,
  - broadcast encryption,
  - witness encryption.

- They developed a framework of Cryptographic Invariant Maps (CIMs), which is
  - ➢ a new primitive closely related to a cryptographic multilinear map, but
  - ➢ whose range does not necessarily have a group structure.

e_n: $X \times \cdots \times X \rightarrow S$ with
- group action $G \curvearrowright X$
- multilinearity w.r.t. the action
- S is not necessarily a group

Cryptographic Invariant Map (CIM) [BGK+18]

- On the top of HHS given by (G, X), we build the notion of Cryptographic Invariant Map (CIM).

- By a CIM we mean a randomized algorithm MapGen that inputs a security parameter λ, outputs public parameters $pp := (X, S, G, e)$, and runs in time polynomial in λ, where
  - (G, X) is a HHS, and S is a set,
  - e is a deterministic algorithm that runs in time polynomial in λ and n, s.t. for $n > 0$, it computes $e_n : X^n \to S$ that satisfies:
    - Invariance property of e_n: for all $x \in X$ and $g_1, \ldots, g_n \in G$, $e_n(g_1 * x, \ldots, g_n * x) = e_n\big((g_1 \cdots g_n) * x, x, \ldots, x\big)$;
    - Non-degeneracy of e_n: for all i with $1 \leq i \leq n$ and $x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n \in X$, the map $X \to S$ defined by $y \mapsto e_n(x_1, \ldots, x_{i-1}, y, x_{i+1}, \ldots, x_n)$ is injective.

Cryptographic Invariant Map (II)

- Fundamental Requirements for Cryptographic Invariant Maps (CIMs):
  The following isomorphism between abelian varieties holds

$$(a_1 * E) \times \cdots \times (a_n * E) \cong (a_1 \cdots a_n) * E \times E^{n-1}$$

  However, we have not yet suitable isomorphism invariants, s.t.
  if $E_1 \times \cdots \times E_n \cong E_1' \times \cdots \times E_n' \Rightarrow e_n(E_1 \times \cdots \times E_n) = e_n(E_1' \times \cdots \times E_n')$,
  if $E_1 \times \cdots \times E_n \ncong E_1' \times \cdots \times E_n' \Rightarrow e_n(E_1 \times \cdots \times E_n) \neq e_n(E_1' \times \cdots \times E_n')$.

- Candidates for Cryptographic Invariant Maps (CIMs) [BGK+18]:
  Seeking for a higher dimensional analog of elliptic curve j-invariant

  - theta null invariant
  - Igusa invariants
  - Invariants of Kummer surfaces

  } not satisfy the fundamental requirements

  - Deligne invariant → not compute efficiently
    (if it's efficiently computable, the isogeny problem become easy !!)

n-way DH Assumptions [BGK+18]

- We say that MapGen satisfies the n-way computational Diffie-Hellman assumption (n-CDH) if for every polynomial time quantum algorithm $\mathcal{S}$,

$$\mathrm{Adv}_{\mathcal{S}}^{n\mathrm{CDH}}(\lambda) := \Pr\left[\mathcal{S}(pp, g_1 * x, \dots, g_n * x) = e_{n-1}\left((g_1 \cdots g_n) * x, x, \dots, x\right)\right]$$

is a negligible function of λ, when $pp \leftarrow \mathrm{MapGen}(\lambda)$, $g_1, \cdots, g_n \leftarrow G$ and $x \leftarrow X$.

- Two distributions $\mathcal{D}_0$ and $\mathcal{D}_1$, where $pp \leftarrow \mathrm{MapGen}(\lambda)$, $g_1, \cdots, g_n \leftarrow G$ and $x \leftarrow X$:
  - $\mathcal{D}_0$ is $(pp, g_1 * x, \dots, g_n * x, s_0)$ where $s_0 = e_{n-1}\left((g_1 \cdots g_n) * x, x, \dots, x\right)$,
  - $\mathcal{D}_1$ is $(pp, g_1 * x, \dots, g_n * x, s_1)$ where s_1 is random in $\mathrm{Im}(e_{n-1}) \subseteq S$.

We say that MapGen satisfies the n-way decisional Diffie-Hellman assumption (n-DDH) if for every polynomial time quantum algorithm $\mathcal{S}$,

$$\mathrm{Adv}_{\mathcal{S}}^{n\mathrm{DDH}}(\lambda) := |\Pr[\,\mathcal{S}(z) = 1 \mid z \leftarrow \mathcal{D}_0\,] - \Pr[\,\mathcal{S}(z) = 1 \mid z \leftarrow \mathcal{D}_1\,]|$$

is a negligible function of λ.

n-way Gap DH Assumption

- We say that MapGen satisfies the n-way gap Diffie-Hellman assumption (n-GDH) if for every polynomial time quantum algorithm $\mathcal{S}$ which accesses the n-DDH oracle $O(\cdot) = n\text{-DDH}(\cdot)$,

$$\mathrm{Adv}_{\mathcal{S}}^{n\mathrm{GDH}}(\lambda) := \Pr\left[\mathcal{S}^{O}(pp, g_1 * x, \dots, g_n * x) = e_{n-1}\left((g_1 \cdots g_n) * x, x, \dots, x\right)\right]$$

is a negligible function of λ, when $pp \leftarrow \mathrm{MapGen}(\lambda)$, $g_1, \cdots, g_n \leftarrow G$ and $x \leftarrow X$. For any input $(x_1', \dots, x_n', s')$ where $x_i' = g_i' * x$ $(i = 1, \dots, n)$, the n-DDH oracle $O(\cdot) = n\text{-DDH}(\cdot)$ acts as follows:

  - $n\text{-DDH}(pp, x_1', \dots, x_n', , s') = 0$ if $s' = e_{n-1}\left((g_1' \cdots g_n') * x, x, \dots, x\right)$,
  - $n\text{-DDH}(pp, x_1', \dots, x_n', , s') = 1$ otherwise.

- Galbraith and Vercauteren showed an attack against the 2-way gap Diffie-Hellman problem on SIDH since the degrees of isogenies used are fixed by public param. as $\ell_1^{e_1}$ and $\ell_2^{e_2}$ for small primes ℓ_1, ℓ_2, e.g., $\ell_1 = 2, \ell_2 = 3$.

As the CSIDH protocol uses random degrees consisting multiple primes $\ell_1, \dots, \ell_n$ and they are not fixed by public parameters, the attack against GDH problem cannot be applied to the CSIDH setting.

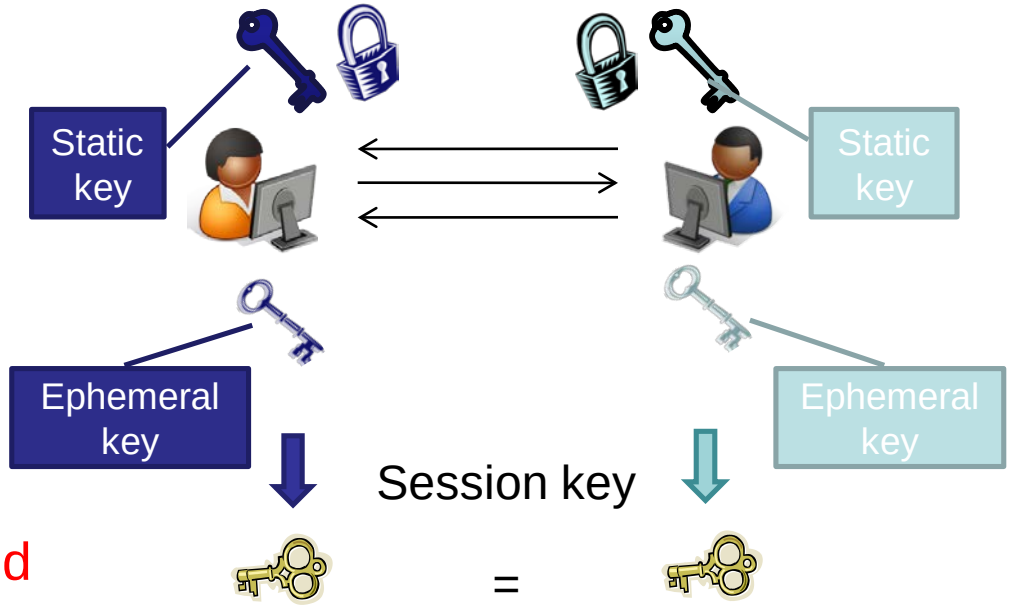
One-Round Authenticated Group Key Exchange (AGKE) on Cryptographic Invariant Maps (CIM)

Authenticated KE (AKE)

- In an AKE protocol, two parties, called initiator and responder, have own static public keys, exchange ephemeral public keys, and compute a session key based on the public keys and the related secret keys.

- Among several security models, the Canetti-Krawczyk (CK) model was proposed to capture leakage of the session state.

- CK^+ model: based on the CK model, several additional security requirements; key compromise impersonation (KCI), weak perfect forward secrecy (wPFS) and maximal exposure attacks (MEX) are integrated.



- We have no QROM secure nor CK^+ secure one-round HHS-based AKE protocols.
  Cf. [DKS18] for CRS-based one

Authenticated Group KE (AGKE)

- It is natural to extend two-party authenticated key exchange to n-party authenticated key exchange for $n > 2$.
- Several security models for AGKE:
  - The G-CK model is an AGKE variant of the CK model, and it captures leakage of the session state.
  - The G-CK$^+$ model integrates the G-CK model with KCI, wPFS and MEX requirements.

- Previously, tripartite AGKE protocols secure in the G-CK or G-CK$^+$ model were given by Manulis et al. and Suzuki et al.
- Li and Yang introduced one-round AGKE protocol from multilinear maps, and Lan et al. introduced one-round AGKE protocol from iO.
  These protocols are not proved in the G-CK or G-CK$^+$ model, and quantum-resistance is not considered.
- Thus, we have no one-round AGKE protocols for general n parties ($n > 3$) secure in the G-CK or G-CK$^+$ model, additionally against quantum adversaries.

Our Contributions from CIM and HHS

- We propose two one-round AGKE protocols on CIMs.
  - n-UM (n-Unified Model) protocol which satisfies the G-CK security: The security is proved under the n-way DDH assumption in the quantum random oracle model (QROM).
  - BC n-DH (biclique n-Diffie-Hellman) protocol which satisfies the G-CK$^+$ security: The security is proved under the n-way GDH assumption in the random oracle model.
- We Instantiate the proposed protocols on HHS with limitation where the number of the user group is two.
  - The CSIDH-based protocols are currently more realistic than the general n-party CIM-based ones due to its implementability.

n-UM Protocol

$T_1 = t_1 * x$	$\cdots$	$T_i = t_i * x$	$\cdots$	$T_n = t_n * x$
$R_1 = r_1 * x$	$\cdots$	$R_i = r_i * x$	$\cdots$	$R_n = r_n * x$
	$\xrightarrow{R_1}$ $\cdots$ $\xleftarrow{R_i}$		$\xrightarrow{R_i}$ $\cdots$ $\xleftarrow{R_n}$	

$$Z_1 = e_{n-1}(T_1, \dots, T_{i-1}, t_i * T_{i+1}, T_{i+2}, \dots, T_n)$$
$$Z_2 = e_{n-1}(R_1, \dots, R_{i-1}, r_i * R_{i+1}, R_{i+2}, \dots, R_n)$$
$$SK = H(\Pi, U_1, \dots, U_n, R_1, \dots, R_n, Z_1, Z_2)$$

- **Public parameters:**
  Protocol identifier $\Pi :=$ nUM.
  A random $(n-1)$-way cryptographic invariant map $(X, S, G, e_{n-1}) \leftarrow_R \mathrm{MapGen}(1^\lambda)$, and random $x \leftarrow_R X$.
  $H: \{0,1\}^* \to \{0,1\}^\lambda$: hash function. Public parameters = $(\Pi, X, S, G, e_{n-1}, x, H)$.

- **Static secret and public keys:** Party U_i chooses $t_i \leftarrow_R G$ as the SSK.
  Then, U_i computes $T_i = t_i * x$ as the SPK.

- **Key exchange:** We suppose a session executed by $\mathbf{U} = (U_1, \dots, U_n)$.
  - U_i chooses $r_i \leftarrow_R G$ as the ESK, and computes $R_i = r_i * x$ as the EPK.
    Then, U_i broadcasts $(\Pi, \mathrm{role}_{i'}, U_i, R_i)$ to $\mathbf{U} \setminus \{U_i\}$.
  - On receiving $(\Pi, \mathrm{role}_{j'}, U_j, R_j)$ for all $j \neq i$, U_i computes
    $Z_1 = e_{n-1}(T_1, \dots, T_{i-1}, t_i * T_{i+1}, T_{i+2}, \dots, T_n)$ and
    $Z_2 = e_{n-1}(R_1, \dots, R_{i-1}, r_i * R_{i+1}, R_{i+2}, \dots, R_n)$.
    U_i generates the session key $SK = H(\Pi, U_1, \dots, U_n, R_1, \dots, R_n, Z_1, Z_2)$.

$$Z_1 = e_{n-1}(T_1, \dots, T_{i-1}, t_i * T_{i+1}, T_{i+2}, \dots, T_n) = e_{n-1}\big((t_1 \cdots t_n) * x, x, \dots, x\big),$$
$$Z_2 = e_{n-1}(R_1, \dots, R_{i-1}, r_i * R_{i+1}, R_{i+2}, \dots, R_n) = e_{n-1}\big((r_1 \cdots r_n) * x, x, \dots, x\big).$$

Security of n-UM Protocol

Theorem (n-UM)

The n-UM protocol is a post-quantum G-CK-secure n-party authenticated key exchange protocol from the n-DDH assumption in the QROM.

In particular, for any quantum adversary $\mathcal{A}$ against the n-UM protocol that runs in time at n_w, involves at most n_u honest parties and activates at most n_s sessions, and makes at most n_h queries to the quantum random oracle and n_q StaticReveal queries,

there exists a n-DDH quantum solver $\mathcal{S}$ such that

$$\mathrm{Adv}^{n\mathrm{DDH}}_{\mathcal{S}}(\lambda) \geq \frac{2 \cdot \mathrm{Adv}^{\mathrm{gck}}_{n\mathrm{UM},\mathcal{A}}(\lambda)^2}{n_u^2 n_s^2 \left(8 n_h n_q + 3\left(n_h + n_q + 1\right)^4\right)},$$

where $\mathcal{S}$ runs in time n_w plus time to perform $O((n_u + n_s)\lambda)$ group operations.

Biclique n−DH Protocol (I)

- **Design Principle:**
  - G-CK$^+$ security: maximum exposure resilience:
    Adv. can obtain either static or ephemeral secret key for each party who is contained in a session.
  - To resist all exposure patterns, parties should compute shared values by all combinations of static or ephemeral key for each party.
  - In n-UM, party U_i computes
    $Z_I = e_{n-1}(T_1, \dots, T_{i-1}, t_i * T_{i+1}, T_{i+2}, \dots, T_n)$ and
    $Z_\emptyset = e_{n-1}(R_1, \dots, R_{i-1}, r_i * R_{i+1}, R_{i+2}, \dots, R_n)$,
    where Z_I (resp. $Z_\emptyset$) consists of only static (resp. ephemeral) keys.
  - Let $I \coloneqq [1, n]$. In biclique n-DH, party U_i additionally computes
    $Z_P = e_{n-1}(V_1, \dots, V_{i-1}, v_i * V_{i+1}, V_{i+2}, \dots, V_n) = e_{n-1}\big((v_1 \cdots v_n) * x, x, \dots, x\big)$
    for all subsets P of I, where
    - $v_i = t_i$ if $i \in P$ and $v_i = r_i$ if $i \notin P$,
    - $V_k = T_k$ if $k \in P$ and $V_k = R_k$ if $k \notin P$ for any $k \neq i$.

    U_i generates the session key $SK = H(\Pi, U_1, \dots, U_n, R_1, \dots, R_n, Z_I, \dots, Z_\emptyset)$.

Biclique n−DH Protocol (II)

- for each subset P of $I = [n]$,

  $Z_P = e_{n-1}(V_1, \ldots, V_{i-1}, v_i * V_{i+1}, V_{i+2}, \ldots, V_n) = e_{n-1}\big((v_1 \cdots v_n) * x, x, \ldots, x\big)$

  where

  - $v_i = t_i$ if $i \in P$ and $v_i = r_i$ if $i \notin P$,
  - $V_k = T_k$ if $k \in P$ and $V_k = R_k$ if $k \notin P$ for any $k \neq i$.

➢ For example, for $n = 8$, and $P = \{1,3,4,8\}$

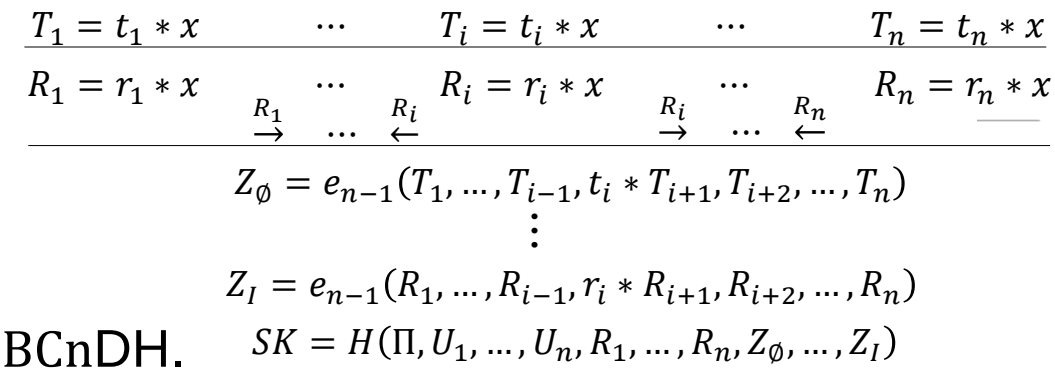
truth value
Tr($i \in P$)

↓

$v_i = t_i$ or r_i

$i = 1$	2	----------	7	8	
1	1	----------	1	1	
0	0	----------	0	0	
t_1	t_2	----------	t_7	t_8	static key
r_1	r_2	----------	r_7	r_8	ephemeral key

Biclique n−DH Protocol (III)



- **Public parameters:**
  We set protocol identifier $\Pi = \mathrm{BCnDH}$.
  The rest is the same as nUM. Public parameters are $(\Pi, X, S, G, e_{n-1}, x, H)$.
- **Static secret and public keys:**
  Same as nUM: (t_i, T_i) are the pair of SSK and SPK of U_i.
- **Key exchange:** we suppose a session executed by $\mathbf{U} = (U_1, \dots, U_n)$.
  - U_i chooses $r_i \leftarrow_R G$ as the ESK, and computes $R_i = r_i * x$ as the EPK. Then, U_i broadcasts $(\Pi, \mathrm{role}_{i'}, U_i, R_i)$ to $\mathbf{U} \setminus \{U_i\}$.
  - On receiving $(\Pi, \mathrm{role}_{j'}, U_j, R_j)$ for all $j \neq i$, U_i computes
    $Z_I = e_{n-1}(T_1, \dots, T_{i-1}, t_i * T_{i+1}, T_{i+2}, \dots, T_n)$, …,
    $Z_\emptyset = e_{n-1}(R_1, \dots, R_{i-1}, r_i * R_{i+1}, R_{i+2}, \dots, R_n)$ as follows:
    for all $P \in \mathcal{P}(I)$, $Z_P = e_{n-1}(V_1, \dots, V_{i-1}, v_i * V_{i+1}, V_{i+2}, \dots, V_n)$, where
    - $v_i = t_i$ if $i \in P$ and $v_i = r_i$ if $i \notin P$,
    - $V_k = T_k$ if $k \in P$ and $V_k = R_k$ if $k \notin P$ for any $k \neq i$.

    Then, U_i generates the session key
    $SK = H(\Pi, U_1, \dots, U_n, R_1, \dots, R_n, Z_I, \dots, Z_\emptyset)$, and completes the session.

Security of biclique n-DH Protocol

Theorem (biclique n-DH)

The biclique n-DH protocol is a post-quantum G-CK$^+$ secure n-party authenticated key exchange protocol from the n-GDH assumption in the ROM.

In particular, for any quantum adversary $\mathcal{A}$ against the biclique n-DH protocol that runs in time at n_w, involves at most n_u honest parties and activates at most n_s sessions, and makes at most n_h queries to the random oracle,

there exists a n-GDH quantum solver $\mathcal{S}$ such that

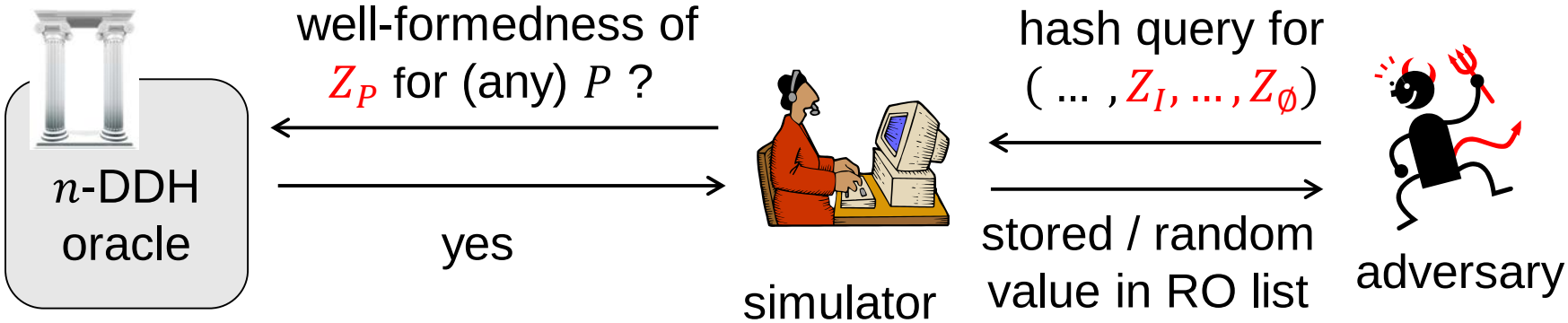
$$\mathrm{Adv}_{\mathcal{S}}^{n\mathrm{GDH}}(\lambda) \geq \min\left\{\frac{1}{n_u^n}, \frac{1}{n_u^{n-1} n_s}, \ldots, \frac{1}{n_u n_s^{n-1}}, \frac{1}{n_s^n}\right\} \cdot \mathrm{Adv}_{\mathrm{BC}n\mathrm{DH},\mathcal{A}}^{\mathrm{gck}^+}(\lambda),$$

where $\mathcal{S}$ runs in time n_w plus time to perform $O((n_u + n_s)\lambda)$ group operations and make $O(n_h + n_s)$ queries to the n-DDH oracle.

Need for n-Gap DH Assumption

- To simulate hash queries and session key reveal queries for all non-trivial combinations, the n-DDH oracle is necessary to keep consistency.
  - If hash function $H(\ \ldots\ , Z_I, \ldots, Z_\emptyset)$ is queried, simulator should check for all subsets P of I, whether Z_P is the form of $Z_P = e_{n-1}(V_1, \ldots, V_{i-1}, v_i * V_{i+1}, V_{i+2}, \ldots, V_n)$ where
    - $v_i = t_i$ if $i \in P$ and $v_i = r_i$ if $i \notin P$,
    - $V_k = T_k$ if $k \in P$ and $V_k = R_k$ if $k \notin P$ for any $k \neq i$.

  by inputting $(V_1, \ldots, V_{i-1}, V_i, V_{i+1}, V_{i+2}, \ldots, V_n)$ to the n-DDH oracle.



CK Secure AKE Protocol from HHS

- We have one-round two-party AKE protocols from HHS as special cases of our AGKE.
  - ➢ 2-UM: HHS-based CK secure AKE protocol

$T_1 = t_1 * x$		$T_2 = t_2 * x$
$R_1 = r_1 * x$	$\overset{R_1}{\rightarrow}$ $\overset{R_2}{\leftarrow}$	$R_2 = r_2 * x$
$Z_1 = t_1 * T_2$		$Z_1 = t_2 * T_1$
$Z_2 = r_1 * R_2$		$Z_2 = r_2 * R_1$

$$SK = H(\Pi, U_1, U_2, R_1, R_2, Z_1, Z_2)$$

Corollary (2-UM)

The 2-UM protocol is a post-quantum CK secure 2-party authenticated key exchange protocol under the 2-DDH assumption in the quantum random oracle model.

CK^+ Secure AKE Protocol from HHS

- Biclique 2-DH: HHS-based CK^+ secure AKE protocol

$T_1 = t_1 * x$		$T_2 = t_2 * x$
$R_1 = r_1 * x$	$\overset{R_1}{\rightarrow}$ $\overset{R_2}{\leftarrow}$	$R_2 = r_2 * x$

$$Z_1 = t_1 * T_2, \quad Z_2 = r_1 * T_2 \qquad Z_1 = t_2 * T_1, \quad Z_2 = t_2 * R_1$$
$$Z_3 = t_1 * R_2, \quad Z_4 = r_1 * R_2 \qquad Z_3 = r_2 * T_1, \quad Z_4 = r_2 * R_1$$

$$SK = H(\Pi, U_1, U_2, R_1, R_2, Z_1, Z_2, Z_3, Z_4)$$

Corollary (biclique 2-DH)

The biclique 2-DH protocol is a post-quantum CK^+ secure 2-party authenticated key exchange protocol under the 2-GDH assumption in the random oracle model.

One-Round Authenticated Key Exchange from SIDH

Previous Works on SIDH-based AKE

- SIDH is considered more efficient than CSIDH key exchange. Therefore, we propose SIDH-based AKE protocols as well.

- Very recently, several SIDH AKE protocols have been proposed. Among them, only Galbraith's proposal achieves one-round AKE, however, the security is proved in the CK and random oracle model.

	security			round complexity
	assumption	model		
SIDH TS2 [Gal18]	SI-CDH	CK	ROM	1-round
AKE-SIDH-SIKE [Lon18]	SI-DDH	CK+	ROM	2-round
LJA [LJA18]	SI-DDH	qCK	QROM	2-round
$AKE_{SIDH\text{-}2}$ [XXW+18]	SI-DDH	CK+	ROM	2-round

Our Contributions on SIDH AKE

- We propose two efficient one-round AKE from SIDH:

  1) **SIDH UM** AKE : secure in CK model under SI-DDH（SI Decisional-DH）assumption in QROM.
  2) **biclique SIDH** AKE : secure in CK+ model under di-SI-GDH（degree-insentitive SI Gap-DH）assumption in ROM.

	security			round complexity
	assumption	model		
SIDH TS2 [Gal18]	SI-CDH	CK	ROM	1-round
AKE-SIDH-SIKE [Lon18]	SI-DDH	CK+	ROM	2-round
LJA [LJA18]	SI-DDH	qCK	QROM	2-round
AKE_{SIDH-2} [XXW+18]	SI-DDH	CK+	ROM	2-round
SIDH UM [FTTY18]	SI-DDH	CK	QROM	1-round
biclique SIDH [FTTY18]	di-SI-GDH	CK+	ROM	1-round

- We introduce a new di-SI-GDH assumption, a variant of SI-GDH assumption, for avoiding the GV (Galbraith-Vercauteren)-type attack for SI-GDH problem.

SIDH Key Exchange

- public parameters: prime p (s.t. $p \pm 1 = f \cdot \ell_A^{e_A} \ell_B^{e_B}$), e.g., $\ell_A = 2, \ell_B = 3$

  supersingular EC $E(\mathbb{F}_{p^2}) \simeq (\mathbb{Z}/(p \pm 1)\mathbb{Z})^2 \supseteq \left(\mathbb{Z}/\ell_A^{e_A}\mathbb{Z}\right)^2 \oplus \left(\mathbb{Z}/\ell_B^{e_B}\mathbb{Z}\right)^2$

  generators $E[\ell_A^{e_A}] = \langle P_A, Q_A \rangle, \quad E[\ell_B^{e_B}] = \langle P_B, Q_B \rangle,$

Alice	Bob
$k_A \leftarrow \mathbb{Z}/\ell_A^{e_A}\mathbb{Z},$	$k_B \leftarrow \mathbb{Z}/\ell_B^{e_B}\mathbb{Z},$
$R_A := P_A + k_A Q_A,$	$R_B := P_B + k_B Q_B,$
$\phi_A : E \to E_A := E/\langle R_A \rangle$	$\phi_B : E \to E_B := E/\langle R_B \rangle,$
$\phi_A(P_B), \phi_A(Q_B) \in E_A$	$\phi_B(P_A), \phi_B(Q_A) \in E_B$
$E_A, \phi_A(P_B), \phi_A(Q_B)$	$E_B, \phi_B(P_A), \phi_B(Q_A)$
$\phi_B(R_A) = \phi_B(P_A) + k_A \phi_B(Q_A)$	$\phi_A(R_B) = \phi_A(P_B) + k_B \phi_A(Q_B)$
$K_{\text{Alice}} := j(E_B/\langle \phi_B(R_A) \rangle)$	$K_{\text{Bob}} := j(E_A/\langle \phi_A(R_B) \rangle)$

- shared key:

$$K_{\text{Alice}} = j(E_B/\langle \phi_B(R_A) \rangle) = j(E/\langle R_A, R_B \rangle) = j(E_A/\langle \phi_A(R_B) \rangle) = K_{\text{Bob}}$$

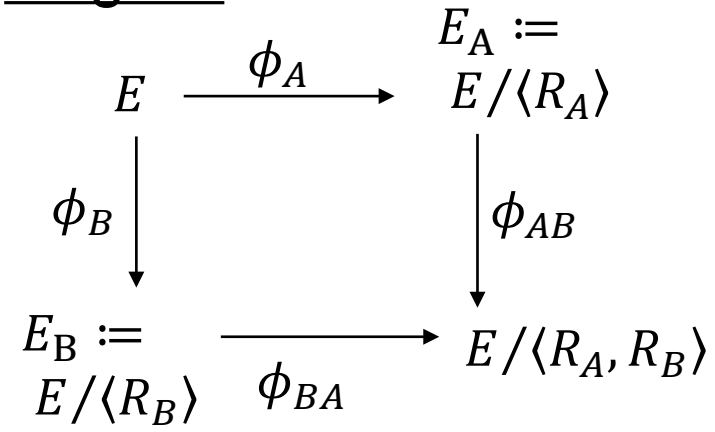
- security: based on the intractability of computing R_A from $E, E_A, \phi_A(P_B), \phi_A(Q_B)$

Crypto-Friendly Notation for SIDH

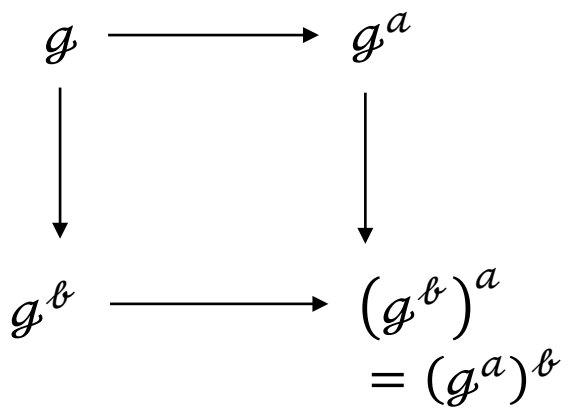
- alternative description for simple presentations of our proposals

- $g := (E; P_A, Q_A, P_B, Q_B)$, $e := (\ell_A, \ell_B, e_A, e_B)$,
- $a := k_A$ $b = k_B$,
- $g^a := (E_A; \phi_A(P_B), \phi_A(Q_B))$, $g^b := (E_B; \phi_B(P_A), \phi_B(Q_A))$,
- $(g^b)^a := j(E_B/\langle \phi_B(R_A) \rangle)$, $(g^a)^b := j(E_A/\langle \phi_A(R_B) \rangle)$.

Original



Crypto-friendly



Proposed Biclique SIDH AKE Protocol

$SK_1 := \mathbb{Z}/\ell_1^{e_1}\mathbb{Z}, \quad SK_2 := \mathbb{Z}/\ell_2^{e_2}\mathbb{Z},$

Alice $a_1 \xleftarrow{U} SK_1, a_2 \xleftarrow{U} SK_2,$ **Bob** $b_1 \xleftarrow{U} SK_1, b_2 \xleftarrow{U} SK_2,$

calculate $A_1 := g^{a_1}, A_2 := g^{a_2}$ calculate $B_1 := g^{b_1}, B_2 := g^{b_2}$

static key of Alice static key of Bob

---

initiator **responder**

$x \xleftarrow{U} SK_1$, calculate $X := g^x$ $y \xleftarrow{U} SK_2$, calculate $Y := g^y$

ephemeral key of Alice ephemeral key of Bob

Y X

use only static keys a_1, B_2 → $Z_1 := Y^{a_1}, Z_2 := B_2{}^x,$ $Z_3 := B_2{}^{a_1}, Z_4 := Y^x,$

$Z_1 := A_1{}^y, Z_2 := X^{b_2},$ $Z_3 := A_1{}^{b_2}, Z_4 := X^y,$ ← use only static keys b_2, A_1

$K_{\text{Alice}} := K_{\text{Bob}} := H(\Pi, Z_1, Z_2, Z_3, Z_4, \text{Alice}, \text{Bob}, X, Y)$

Two Types of SI-Gap-DH Problem (I)

ds-, di-SI-GDH problems

Let $\mathcal{S}$ be a quantum machine adversary.

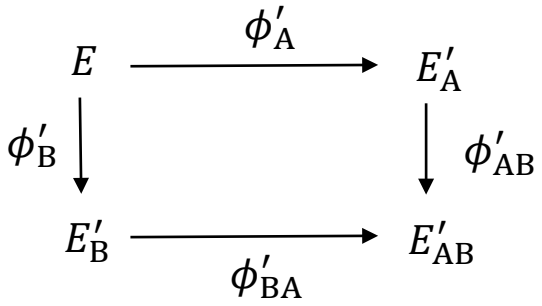
For $\mathrm{pk}^{\mathrm{sidh}} = (\mathcal{g} := (E; P_{\mathrm{A}}, Q_{\mathrm{A}}, P_{\mathrm{B}}, Q_{\mathrm{B}}), e := (\ell_{\mathrm{A}}, \ell_{\mathrm{B}}, e_{\mathrm{A}}, e_{\mathrm{B}})) \xleftarrow{\mathrm{R}} \mathrm{Gen}^{\mathrm{sidh}}(1^{\lambda})$ and $a \xleftarrow{\mathrm{U}} SK_{\mathrm{A}}, b \xleftarrow{\mathrm{U}} SK_{\mathrm{B}}$, $\mathcal{S}$ receives $(\mathrm{pk}^{\mathrm{sidh}}, \mathcal{g}^{a}, \mathcal{g}^{b})$, and $\mathcal{S}$ accesses the SI-DDH oracle for any input $\mathcal{X} = (\mathrm{pk}^{\mathrm{sidh}}, (E'_{\mathrm{A}}; P'_{\mathrm{AB}}, Q'_{\mathrm{AB}}), (E'_{\mathrm{B}}; P'_{\mathrm{BA}}, Q'_{\mathrm{BA}}), \mathcal{h}')$ where $P'_{\mathrm{AB}}, Q'_{\mathrm{AB}}$ (resp. $P'_{\mathrm{BA}}, Q'_{\mathrm{BA}}$) are points in $E'_{\mathrm{A}}(\mathbb{F}_{p^2})$ (resp. $E'_{\mathrm{B}}(\mathbb{F}_{p^2})$) and $\mathcal{h}' \in \mathbb{F}_{p^2}$, and then outputs $\mathcal{h} \in \mathbb{F}_{p^2}$. If $\mathcal{h} = (\mathcal{g}^{a})^{b} \left(= (\mathcal{g}^{b})^{a}\right)$, $\mathcal{S}$ wins.

According to the behavior of SI-DDH oracle, we have the following two types of SI-GDH problems, i.e.,

- degree-sensitive SI-GDH (ds-GDH) problem and
- degree-insensitive SI-GDH (di-GDH) problem.

Two Types of SI-Gap-DH Problem (II)

- We have two types of SI-GDH problems according to the SI-DDH oracle.
  - ➢ degree-sensitive SI-GDH (ds-GDH) problem:
    The ds-DDH oracle answers true if
    there exists a supersingular EC E'_{AB} and a commutative diagram below s.t.
    - degree d'_{A} of ϕ'_{A} (and ϕ'_{BA}) satisfies $d'_{\mathrm{A}} = \ell_{\mathrm{A}}^{e_{\mathrm{A}}}$ and
      degree d'_{B} of ϕ'_{B} (and ϕ'_{AB}) satisfies $d'_{\mathrm{B}} = \ell_{\mathrm{B}}^{e_{\mathrm{B}}}$
    - $P'_{\mathrm{AB}} = \phi'_{\mathrm{A}}(P_{\mathrm{B}}), Q'_{\mathrm{AB}} = \phi'_{\mathrm{A}}(Q_{\mathrm{B}}), P'_{\mathrm{BA}} = \phi'_{\mathrm{B}}(P_{\mathrm{A}}), Q'_{\mathrm{BA}} = \phi'_{\mathrm{B}}(Q_{\mathrm{A}})$ and $\mathcal{h}' = j(E'_{\mathrm{AB}})$.
  - ➢ degree-insensitive SI-GDH (di-GDH) problem:
    The di-DDH oracle answers true if
    there exists a supersingular EC E'_{AB} and a commutative diagram below s.t.
    - degree d'_{A} of ϕ'_{A} (and ϕ'_{BA}) is a power of ℓ_{A} and
      degree d'_{B} of ϕ'_{B} (and ϕ'_{AB}) is a power of ℓ_{B}
    - $P'_{\mathrm{AB}} = \phi'_{\mathrm{A}}(P_{\mathrm{B}}), Q'_{\mathrm{AB}} = \phi'_{\mathrm{A}}(Q_{\mathrm{B}}), P'_{\mathrm{BA}} = \phi'_{\mathrm{B}}(P_{\mathrm{A}}), Q'_{\mathrm{BA}} = \phi'_{\mathrm{B}}(Q_{\mathrm{A}})$ and $\mathcal{h}' = j(E'_{\mathrm{AB}})$.



$$d'_{\mathrm{A}} = \deg(\phi'_{\mathrm{A}}) = \deg(\phi'_{\mathrm{BA}})$$

$$d'_{\mathrm{B}} = \deg(\phi'_{\mathrm{B}}) = \deg(\phi'_{\mathrm{AB}})$$

Galbraith-Vercauteren (GV) Attack

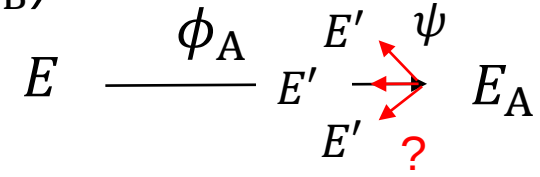
- Attack for SIDH isogeny problem using the decision degree (DD) oracle for SIDH isogenies.

  - Solver $\mathcal{S}$ obtains the following SIDH isogeny problem instance

$$(\,\mathrm{pk}^{\mathrm{sidh}} = (\mathcal{g} := (\,E; P_{\mathrm{A}}, Q_{\mathrm{A}}, P_{\mathrm{B}}, Q_{\mathrm{B}}), e := (\,\ell_{\mathrm{A}}, \ell_{\mathrm{B}}, e_{\mathrm{A}}, e_{\mathrm{B}}\,)),$$
$$E_{\mathrm{A}}, P_{\mathrm{AB}} := \phi_{\mathrm{A}}(P_{\mathrm{B}}), Q_{\mathrm{AB}} := \phi_{\mathrm{A}}(Q_{\mathrm{B}}))$$

    where $\phi_A: E \to E_A$ is an $\ell_{\mathrm{A}}^{e_{\mathrm{A}}}$-isogeny, and the goal of $\mathcal{S}$ is to reveal ϕ_{A}.

  - $\mathcal{S}$ calculates $u \in \mathbb{Z}$ such that $u \cdot \ell_{\mathrm{A}} \equiv 1 \pmod{\ell_{\mathrm{B}}}$ and then ℓ_{A}-isogeny $\psi: E_{\mathrm{A}} \to E'$. $\mathcal{S}$ sends



$$(\,\widetilde{\mathrm{pk}}^{\mathrm{sidh}} = (\mathcal{g}, \tilde{e} := (\,\ell_{\mathrm{A}}, \ell_{\mathrm{B}}, e_{\mathrm{A}} - 1, e_{\mathrm{B}}\,),$$
$$E', u \cdot \psi(P_{\mathrm{AB}}), u \cdot \psi(Q_{\mathrm{AB}}))$$

    to the DD oracle. The oracle distinguishes whether E' is $\ell_{\mathrm{A}}^{e_{\mathrm{A}}-1}$-isogenous to E or $\ell_{\mathrm{A}}^{e_{\mathrm{A}}+1}$- isogenous to E, i.e., whether ψ is a backtracking step on the path given by ϕ_A. By repeating this, $\mathcal{S}$ reveals ϕ_{A}.

Solving ds-SI-GDH Problem by GV-type Attack

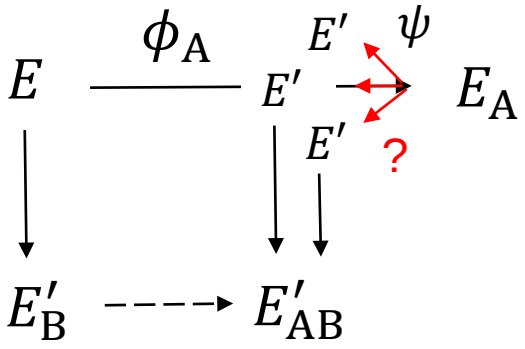
- Attack for ds-SI-GDH problem using ds-SI-DDH oracle.

  - Solver $\mathcal{S}$ obtains the following ds-SI-GDH problem instance.

$$(\mathrm{pk}^{\mathrm{sidh}} = (\mathcal{g} := (E; P_{\mathrm{A}}, Q_{\mathrm{A}}, P_{\mathrm{B}}, Q_{\mathrm{B}}), e := (\ell_{\mathrm{A}}, \ell_{\mathrm{B}}, e_{\mathrm{A}}, e_{\mathrm{B}})),$$
$$E_{\mathrm{A}}, P_{\mathrm{AB}} := \phi_{\mathrm{A}}(P_{\mathrm{B}}), Q_{\mathrm{AB}} := \phi_{\mathrm{A}}(Q_{\mathrm{B}}), \dots)$$

  where $\phi_A: E \to E_A$ is an $\ell_{\mathrm{A}}^{e_{\mathrm{A}}}$-isogeny, and the goal of $\mathcal{S}$ is to reveal ϕ_{A}.

  - $\mathcal{S}$ calculates $u \in \mathbb{Z}$ s.t. $u \cdot \ell_{\mathrm{A}} \equiv 1 \pmod{\ell_{\mathrm{B}}}$ and then ℓ_{A}-isogeny $\psi: E_{\mathrm{A}} \to E'$, and $\ell_{\mathrm{B}}^{e_{\mathrm{B}}}$-isogenies $E \to E'_{\mathrm{B}}$, $E' \to E'_{\mathrm{AB}}$ s.t. the right diagram is commutative. $\mathcal{S}$ sends



$$(\widetilde{\mathrm{pk}}^{\mathrm{sidh}} = (\mathcal{g}, \tilde{e} := (\ell_{\mathrm{A}}, \ell_{\mathrm{B}}, e_{\mathrm{A}} - 1, e_{\mathrm{B}}),$$
$$E', E'_{\mathrm{B}}, \dots, j(E'_{\mathrm{AB}}))$$

  to the ds-SI-DDH oracle. The oracle distinguishes whether ψ is a backtracking step on the path given by ϕ_A. By repeating this, $\mathcal{S}$ reveals ϕ_{A}.

Security of Biclique SIDH AKE from di-SI-GDH Assumption

Theorem

The biclique SIDH protocol is a post-quantum CK^+-secure authenticated key exchange protocol under the di-SI-GDH assumption in the random oracle model.

- To distinguish the degree of isogeny (or distance between two elliptic curves in the ℓ_A-isogeny graph) is crucial for the GV-type attack.
- The GV-type attack adversaries have no advantages in the di-SI-GDH problem, where the decision oracle cannot distinguish the degree of isogeny.
- Therefore, (in contrast to the ds-SI-GDH problem,) we may assume that the di-SI-GDH problem cannot be solved by any efficient adversaries, and can be used for the basis of the security of our biclique scheme.

References

[FTTY18] A. Fujioka, K. Takashima, S. Terada, K. Yoneyama, "Supersingular Isogeny Diffie-Hellman Authenticated Key Exchange", to appear in ICISC 2018, full version: ePrint Archive 2018/730

[FTY18] A. Fujioka, K. Takashima, K. Yoneyama, "One-Round Authenticated Group Key Exchange from Isogenies", ePrint Archive 2018/1033

Conclusions

- We proposed two one-round AGKE protocols from Cryptographic Invariant Maps (CIMs).
- We instantiate the proposed protocols on HHS with limitation where the number of the user group is two.

  In particular, the protocols instantiated by CSIDH are currently more realistic than the general n-party CIM-based ones.
- We also proposed two one-round AKE protocols from SIDH, which is more efficient.

Thank you for your attention